# Liquid Tensor Experiment

Ab is nice

# TopAb is ugly

$$\mathbb{R}^{\delta} \to \mathbb{R}$$

AbSh($X$) is nice

# Profinite sets

# Condensed sets

$$\mathrm{CompHaus} \hookrightarrow \mathrm{Cond}(\mathrm{Set})$$

# Cond(Ab) is nice

# Condensed rings/modules

# Six functor formalism

# Analytic rings

# Analytic geometry

# Applications

# Real analysis

$$\{0.d_1d_2d_3\cdots \mid d_i = 0, 1, \ldots, 9\}$$

# Scholze’s challenge

# Some details

# Definition

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$$S \mapsto \mathcal{A}[S]$$

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satisfying some conditions.

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$$\mathcal{M}_{<p}(S) = \varinjlim_{p' < p} \mathcal{M}_{p'}(S)$$

# Main theorem

Fix $0 < p \leq 1$.

Then $(\mathbb{R}, \mathcal{M}_{<p})$ is an analytic ring.

Functional analysis

Homological algebra

Theorem 9.5

$\mathbb{Z}((T)) \to \mathbb{R}$

Condensed mathematics

Categorical reduction steps

Main theorem

# Breen–Deligne resolution

Breen–Deligne resolution

Polyhedral lattices

Breen–Deligne resolution

Polyhedral lattices

Normed exactness

Breen–Deligne resolution

Polyhedral lattices

Normed exactness

Chase inequalities through spectral sequences

# Joint work with

- Peter Scholze

The Lean community

- Damiano Testa
- Patrick Massot
- Kevin Buzzard
- Riccardo Brasca
- Adam Topaz
- Scott Morrison