

Liquid Tensor Experiment

Ab is nice

TopAb is ugly

$$\mathbb{R}^{\delta} \rightarrow \mathbb{R}$$

AbSh(X) is nice

Profinite sets

Condensed sets

$$\mathbf{CompHaus} \hookrightarrow \mathbf{Cond}(\mathbf{Set})$$

Cond(*Ab*) is nice

Condensed rings/modules

Six functor formalism

Analytic rings

Analytic geometry

Applications

Real analysis

$$\{0.d_1d_2d_3\cdots \mid d_i = 0, 1, \dots, 9\}$$

Scholze's challenge

Some details

Definition

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satisfying some conditions.

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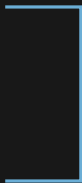
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$$\mathcal{M}_{<p}(S) = \varinjlim_{p' < p} \mathcal{M}_{p'}(S)$$

Main theorem

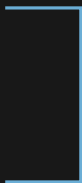
Fix $0 < p \leq 1$.

Then $(\mathbb{R}, \mathcal{M}_{< p})$ is an analytic ring.



Functional analysis
Homological algebra

Theorem 9.5



$$\mathbb{Z}((T)) \rightarrow \mathbb{R}$$

Condensed mathematics
Categorical reduction steps

Main theorem

Breen–Deligne resolution

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Polyhedral lattices

Breen–Deligne resolution

Polyhedral lattices

Normed exactness

Breen–Deligne resolution

Polyhedral lattices

Normed exactness

Chase inequalities through spectral sequences

Joint work with

- ▶ Peter Scholze

The Lean community

- ▶ Damiano Testa
- ▶ Patrick Massot
- ▶ Kevin Buzzard
- ▶ Riccardo Brasca
- ▶ Adam Topaz
- ▶ Scott Morrison