2.6.1. Exercise. In this exercise, we prove **??**, by considering condensed abelian groups as functors

$$\mathrm{Extr}^{\mathrm{op}} \to \mathrm{Ab}$$

that send finite disjoint unions in Extr to finite products in Ab.

Let $i \mapsto M_i \colon I \to \mathrm{Cond}(\mathrm{Ab})$ be a diagram. Show that the limit and colimit of this diagram are given by $S \mapsto \lim_i M_i(S)$ and $S \mapsto \mathrm{colim}_i\, M_i(S)$ respectively. Now deduce the properties of Cond(Ab) from the corresponding properties of Ab. (Hint: use that in Ab finite products are the same as finite coproducts, i.e., they are biproducts. Hence formation of arbitrary (co)limits commutes with finite biproducts.)

2.6.2. Exercise. Show that the subcategory of Cond(Ab) on all compactological abelian groups is stable under all limits and filtered colimits whose transition maps are monomorphisms.

2.6.3. Exercise. Let S be an extremally disconnected set. Show that $\mathbb{Z}[S]$ is a projective object in Cond(Ab).

2.6.4. Exercise. Prove parts of **??**, namely:

(*i*) The map f naturally induces a map $\mathbb{Z}[X]_{\leq n} \to \mathbb{Z}]Y]_{\leq n}$.

(*ii*) If X is finite, then so is $\mathbb{Z}[X]_{\leq n}$.

(*iii*) If X is profinite, then so is $\mathbb{Z}[X]_{\leq n}$. Indeed, if $X = \lim_i X_i$ with X_i finite, then $\mathbb{Z}[X]_{\leq n} = \lim_i \mathbb{Z}[X_i]_{\leq n}$.

2.6.5. Exercise. Let A be a condensed abelian group. Show that A is quasiseparated if and only if $0 \to A$ is closed.

2.6.6. Exercise. Let $0 \to A \to B \to C \to 0$ be a short exact sequence of condensed abelian groups. If A and C are quasiseparated (aka compactological) show that B is also quasiseparated. (Hint: use the preceding exercise.)

2.6.7. Exercise. In the context of **??** prove the identity $d \circ h + h \circ d = \mathrm{id}$ in each degree.