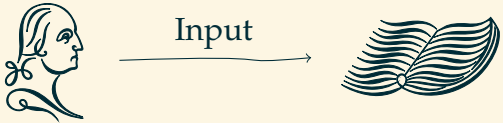
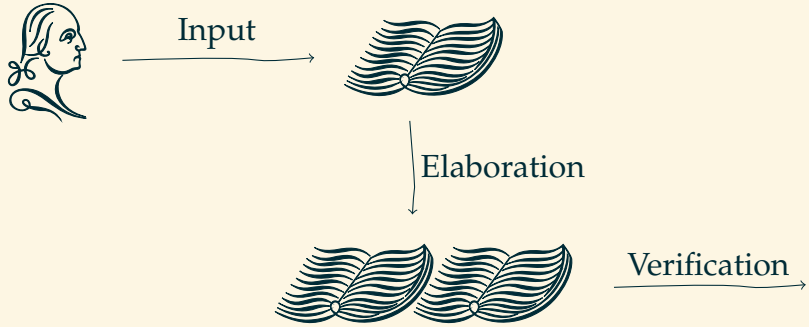
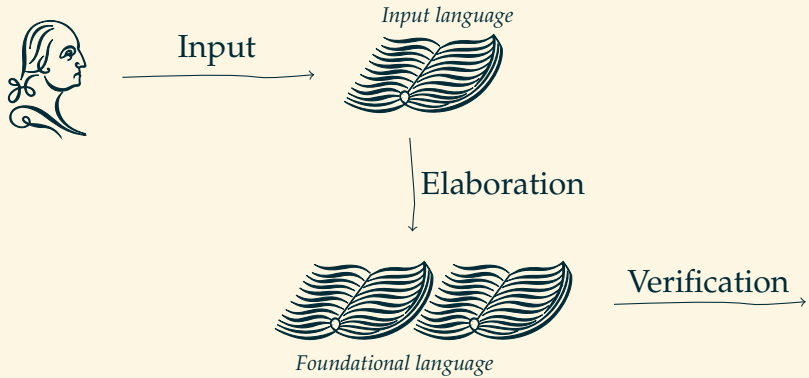
L∃∀N proof assistant


Input

Input

Verification


Input
Elaboration
Verification


Input
Input language
Elaboration
Verification
Foundational language

Foundations

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- I don’t care about foundations

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- Set theory $\iff$ Type theory

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The best of both worlds(?):

Input language:	type-theoretic
Foundation:	set-theoretic

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Example: TypeScript / JavaScript

Naive type theory

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$$X \to Y = \prod_x Y$$

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- Induction principle

Propositions as types

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$$P \to Q \quad \text{“}=\text{”} \quad P \implies Q$$

Some components of elaboration

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- unification

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- type classes

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- tactics

Unification

Example: $f : X \to Y$ is *surjective* if
for all y there exists an x such that $f(x) = y$

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"Where does x live?"

Type class system

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A lemma about topological groups can be applied to $\mathbb{R}$

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A lemma about topological groups can be applied to $\mathbb{R}$

because $\mathbb{R}$ is a normed field.

Example tactic: `simp`

Simp-lemmas have the form $a = b$ or $p \iff q$.

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↓ simp

rbndom exbmqle formulb

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$$1 \cdot x = x, \qquad a - a = 0, \qquad f(a+b) = f(a) + f(b)$$

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Bad examples:

$$p \wedge q \iff q \wedge p, \qquad f(x) = f(y) \iff x - y \in \ker(f)$$

Example tactic: `simp`

`simp` can prove

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$$\int_0^1 x^n \,\mathrm{d}x = \frac{1}{n+1}$$

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- Ad-hoc tactics

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See articles and website of Freek Wiedijk

Mathlib overview

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`leanprover-community.github.io/mathlib-overview.html`

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Are you interested? Join Zulip and chat with us!

Pointers

Community website
`leanprover-community.github.io`

Chatroom
`leanprover.zulipchat.com`

Natural number game
`wwwf.imperial.ac.uk/~buzzard/xena/natural_number_game/`