The combinatorial motivation

Knowing the final result of the classification can be a good motivation for studying it.

Moreover, this classification (or parts of it) have also been found in other parts of mathematics (for example in the classification of singularities on complex algebraic surfaces).

History

1888-96 Sophus Lie, Friedrich Engel, Georg Scheffers
Many great ideas, but needed to be made precise

1888-90 Wilhelm Killing classification / $\mathbb{C}$

1894 Élie Cartan made the theory and classification rigorous

Gelfand ran a seminar on Lie groups.
asked Dynkin to prepare
a survey on the classification.

1944-47 Eugene Dynkin Dynkin found the literature hard to read
and invented **Dynkin diagrams**

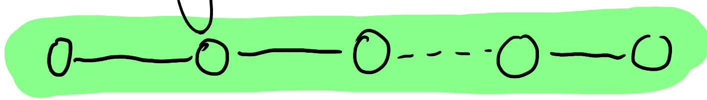
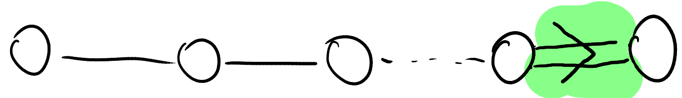
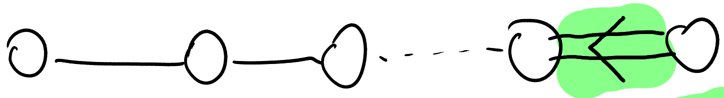
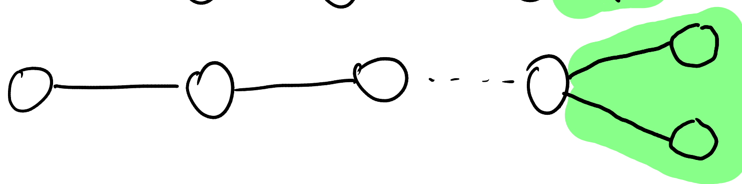
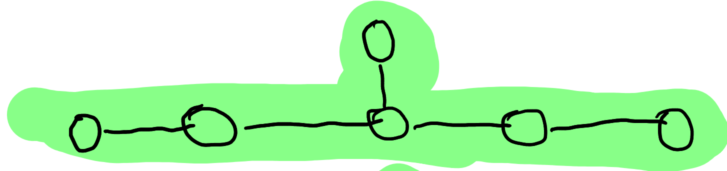
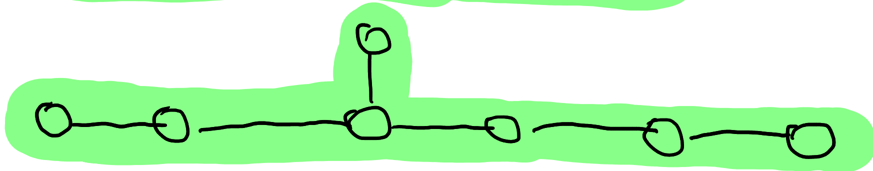
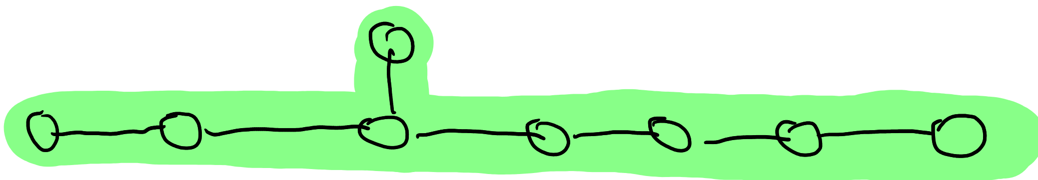
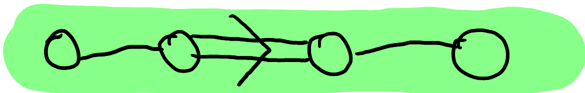
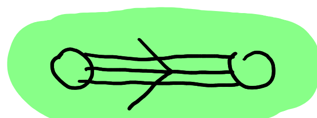
Quoting a quote from Wikipedia

Of Dynkin's 1947 paper "Structure of semisimple Lie algebras",
Bertram Kostant wrote:

In this paper, using only elementary mathematics,
and starting with almost nothing,
Dynkin, brilliantly and elegantly developed
the structure and machinery of semisimple Lie algebras.
What he accomplished in this paper
was to take a hitherto esoteric subject,
and to make it into beautiful and powerful mathematics.

— Bertram Kostant, "Selected papers", p. 363

Dynkin diagrams

	Type	(cond.)	Lie algebra	Diagram
Classical	A_n	$(n \geq 1)$	sl_{n+1}	
	B_n	$(n \geq 2)$	so_{2n+1}	
	C_n	$(n \geq 3)$	sp_{2n}	
	D_n	$(n \geq 4)$	so_{2n}	
Exceptional	E_6	—	?	
	E_7	—	?	
	E_8	—	?	
	F_4	—	?	
	G_2	—	?	

Classification theorem

There is a 1-to-1 correspondence between

$\left\{ \begin{array}{c} \text{isomorphism classes of} \\ \text{fin. dim. simple Lie algebras} \\ \text{over } \mathbb{C} \end{array} \right\}$

and

$\left\{ \text{Dynkin diagrams} \right\}$